# Package: excursions (via r-universe)

August 26, 2024

Type Package

Title Excursion Sets and Contour Credibility Regions for Random Fields

Version 2.5.8.9000

Description Functions that compute probabilistic excursion sets, contour credibility regions, contour avoiding regions, and simultaneous confidence bands for latent Gaussian random processes and fields. The package also contains functions that calculate these quantities for models estimated with the INLA package. The main references for excursions are Bolin and Lindgren (2015) <doi:10.1111/rssb.12055>, Bolin and Lindgren (2017) <doi:10.1080/10618600.2016.1228537>, and Bolin and Lindgren (2018) <doi:10.18637/jss.v086.i05>. These can be generated by the citation function in R.

**Depends** R (>= 3.5), Matrix, sp

**Suggests** INLA (>= 21.08.31), testthat (>= 3.0.0), sf

**Imports** fmesher (>= 0.1.2), graphics, methods, stats, withr

**SystemRequirements** Gnu Scientific Library version >= 2.1

Additional\_repositories https://inla.r-inla-download.org/R/testing

BugReports https://github.com/davidbolin/excursions/issues

License GPL (>=3)

URL https://github.com/davidbolin/excursions

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NeedsCompilation yes

RoxygenNote 7.2.3

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Repository https://davidbolin.r-universe.dev

RemoteUrl https://github.com/davidbolin/excursions

RemoteRef HEAD

**RemoteSha** 8f7b0f974a2754c3a65081de82504c78a54bc809

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# Description

excursions contains functions that compute probabilistic excursion sets, contour credibility regions, contour avoiding regions, contour map quality measures, and simultaneous confidence bands for latent Gaussian random processes and fields. A detailed manual can be found in the paper Bolin, D and Lindgren, F (2018) *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, 86(5), 1–20.

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#### **Details**

The main functions in the package fall into three different categories described below.

# Excursion sets, contour credibility regions, and contour avoiding regions

The main functions for computing excursion sets, contour credibility regions, and contour avoiding regions are

excursions The main function for Gaussian models.

excursions.inla Interface for latent Gaussian models estimated using INLA.

excursions.mc Function for analyzing models that have been estimated using Monte Carlo methods.

The output from the functions above provides a discrete domain estimate of the regions. Based on this estimate, the function continuous computes a continuous domain estimate.

The main reference for these functions is Bolin, D. and Lindgren, F. (2015) *Excursion and contour uncertainty regions for latent Gaussian models*, JRSS-series B, vol 77, no 1, pp 85-106.

### Contour map quality measures

The package provides several functions for computing contour maps and their quality measures. These quality measures can be used to decide on an appropriate number of contours to use for the contour map.

The main functions for computing contour maps and the corresponding quality measures are

contourmap The main function for Gaussian models.

contourmap.inla Interface for latent Gaussian models estimated using INLA.

contourmap.mc Function for analyzing models that have been estimated using Monte Carlo methods.

Other noteworthy functions relating to contourmaps are tricontour and tricontourmap, which compute contour curves for functions defined on triangulations, as well as contourmap.colors which can be used to compute appropriate colors for displaying contour maps.

The main reference for these functions is Bolin, D. and Lindgren, F. (2017) *Quantifying the uncertainty of contour maps*, Journal of Computational and Graphical Statistics, 26:3, 513-524.

#### Simultaneous confidence bands

The main functions for computing simultaneous confidence bands are

```
simconf Function for analyzing Gaussian models.
```

simconf.inla Function for analyzing latent Gaussian models estimated using INLA.

simconf.mc Function for analyzing models estimated using Monte Carlo methods.

simconf.mixture Function for analyzing Gaussian mixture models.

The main reference for these functions is Bolin et al. (2015) *Statistical prediction of global sea level from global temperature*, Statistica Sinica, Vol 25, pp 351-367.

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#### Author(s)

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#### See Also

Useful links:

- https://github.com/davidbolin/excursions
- Report bugs at https://github.com/davidbolin/excursions/issues

continuous

Calculate continuous domain excursion and credible contour sets

# Description

Calculates continuous domain excursion and credible contour sets

## Usage

```
continuous(
  ex,
  geometry,
  alpha,
  method = c("log", "linear", "step"),
  output = c("sp", "fm", "inla"),
  subdivisions = 1,
  calc.credible = TRUE
)
```

## **Arguments**

ex An excurobj object generated by a call to excursions or contourmap.

geometry Specification of the lattice or triangulation geometry of the input. One of list(x, y)

y), list(loc, dims), fm\_lattice\_2d, inla.mesh.lattice, fm\_mesh\_2d, or inla.mesh, where x and y are vectors, loc is a two-column matrix of coordinates, and dims is the lattice size vector. The first three versions are all treated

topologically as lattices, and the lattice boxes are assumed convex.

alpha The target error probability. A warning is given if it is detected that the information ex isn't sufficient for the given alpha. Defaults to the value used when

calculating ex.

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method The spatial probability interpolation transformation method to use. One of log,

linear, or step. For log, the probabilities are interpolated linearly in the trans-

formed scale. For step, a conservative step function is used.

output Specifies what type of object should be generated. sp gives a SpatialPolygons

object, and fm or inla gives a fm\_segm object.

subdivisions The number of mesh triangle subdivisions to perform for the interpolation of

the excursions or contour function. 0 is no subdivision. The setting has a small effect on the evaluation of P0 for the log method (higher values giving higher accuracy) but the main effect is on the visual appearance of the interpolation.

Default=1.

calc.credible Logical, if TRUE (default), calculate credible contour region objects in addition

to avoidance sets.

#### Value

A list with elements

M SpatialPolygons or inla.mesh.segment object. The subsets are tagged, so

that credible regions are tagged "-1", and regions between levels are tagged

as.character(0:nlevels).

F Interpolated F function.

G Contour and inter-level set indices for the interpolation.

F. geometry Mesh geometry for the interpolation.

P0 P0 measure based on interpolated F function (only for contourmap input).

#### Author(s)

Finn Lindgren <finn.lindgren@gmail.com>

#### References

Bolin, D. and Lindgren, F. (2017) *Quantifying the uncertainty of contour maps*, Journal of Computational and Graphical Statistics, vol 26, no 3, pp 513-524.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

```
## Not run:
if (require("fmesher")) {
    # Generate mesh and SPDE model
    n.lattice <- 10 # Increase for more interesting, but slower, examples
    x <- seq(from = 0, to = 10, length.out = n.lattice)
    lattice <- fm_lattice_2d(x = x, y = x)
    mesh <- fm_rcdt_2d_inla(lattice = lattice, extend = FALSE, refine = FALSE)

# Generate an artificial sample
    sigma2.e <- 0.1</pre>
```

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```
n.obs <- 100
 obs.loc <- cbind(</pre>
   runif(n.obs) * diff(range(x)) + min(x),
   runif(n.obs) * diff(range(x)) + min(x)
 Q <- fm_matern_precision(mesh, alpha = 2, rho = 3, sigma = 1)
 x \leftarrow fm_sample(n = 1, Q = Q)
 A <- fm_basis(mesh, loc = obs.loc)
 Y <- as.vector(A %*% x + rnorm(n.obs) * sqrt(sigma2.e))
 ## Calculate posterior
 Q.post <- (Q + (t(A) %*% A) / sigma2.e)
 mu.post <- as.vector(solve(Q.post, (t(A) %*% Y) / sigma2.e))</pre>
 vars.post <- excursions.variances(chol(Q.post))</pre>
 ## Calculate contour map with two levels
 map <- contourmap(</pre>
   n.levels = 2, mu = mu.post, Q = Q.post,
   alpha = 0.1, F.limit = 0.1, max.threads = 1
 ## Calculate the continuous representation
 sets <- continuous(map, mesh, alpha = 0.1)</pre>
 ## Plot the results
 reo <- mesh$idx$lattice</pre>
 cols <- contourmap.colors(map,</pre>
   col = heat.colors(100, 1, rev = TRUE),
   credible.col = grey(0.5, 1)
 names(cols) <- as.character(-1:2)</pre>
 par(mfrow = c(2, 2))
 image(matrix(mu.post[reo], n.lattice, n.lattice),
   main = "mean", axes = FALSE, asp = 1
 image(matrix(sqrt(vars.post[reo]), n.lattice, n.lattice),
   main = "sd", axes = FALSE, asp = 1
 image(matrix(map$M[reo], n.lattice, n.lattice),
   col = cols, axes = FALSE, asp = 1
 idx.M <- setdiff(names(sets$M), "-1")</pre>
 plot(sets$M[idx.M], col = cols[idx.M])
## End(Not run)
```

contourmap

}

Contour maps and contour map quality measures for latent Gaussian models

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## **Description**

contourmap is used for calculating contour maps and quality measures for contour maps for Gaussian models.

#### Usage

```
contourmap(
 mu,
 Q,
  vars,
 n.levels,
  ind,
 levels,
 type = c("standard", "pretty", "equalarea", "P0-optimal", "P1-optimal", "P2-optimal"),
 compute = list(F = TRUE, measures = NULL),
 use.marginals = TRUE,
  alpha,
 F.limit,
 n.iter = 10000,
  verbose = FALSE,
 max.threads = 0.
  seed = NULL
)
```

#### **Arguments**

Expectation vector. mu Precision matrix. Q vars Precomputed marginal variances (optional). n.levels Number of levels in contour map. ind Indices of the nodes that should be analyzed (optional). levels Levels to use in contour map. Type of contour map. One of: type 'standard' Equidistant levels between smallest and largest value of the posterior mean (default). 'pretty' Equally spaced 'round' values which cover the range of the values in the posterior mean. 'equalarea' Levels such that different spatial regions are approximately equal in size. 'P0-optimal' Levels chosen to maximize the P0 measure. **'P1-optimal'** Levels chosen to maximize the P1 measure. **'P2-optimal'** Levels chosen to maximize the P2 measure. compute A list with quality indices to compute

computed (default TRUE).

'F': TRUE/FALSE indicating whether the contour map function should be

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'measures': A list with the quality measures to compute ("P0", "P1", "P2") or corresponding bounds based only on the marginal probabilities ("P0-bound", "P1-bound", "P2-bound").

use.marginals Only marginal distributions are used when finding P-optimal maps (default TRUE).

alpha Maximal error probability in contour map function (default=1).

F.limit The limit value for the computation of the F function. F is set to NA for all

nodes where F<1-F.limit. Default is F.limit = alpha.

n.iter Number or iterations in the MC sampler that is used for calculating the quantities

in compute. The default value is 10000.

verbose Set to TRUE for verbose mode (optional).

max.threads Decides the number of threads the program can use. Set to 0 for using the

maximum number of threads allowed by the system (default).

seed Random seed (optional).

#### **Details**

The Gaussian model is specified using the mean mu and the precision matrix Q. The contour map is then computed for the mean, using either the contour levels specified in levels, or n.levels contours that are placed according to the argument type.

A number of quality measures can be computed based based on the specified contour map and the Gaussian distribution. What should be computed is specified using the compute argument. For details on these quanties, see the reference below.

#### Value

contourmap returns an object of class "excurobj" with the following elements

u Contour levels used in the contour map.

n.levels The number of contours used.

u.e The values associated with the level sets G\_k.

G A vector which shows which of the level sets G\_k each node belongs to.

map Representation of the contour map with map[i]=u.e[k] if i is in  $G_k$ .

F The contour map function (if computed).

M Contour avoiding sets (if F is computed). M = -1 for all non-significant nodes

and M = k for nodes that belong to  $M_k$ .

P0/P1/P2 Calculated quality measures (if computed).

P0bound/P1bound/P2bound

Calculated upper bounds quality measures (if computed).

meta A list containing various information about the calculation.

#### Author(s)

David Bolin <davidbolin@gmail.com>

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#### References

Bolin, D. and Lindgren, F. (2017) *Quantifying the uncertainty of contour maps*, Journal of Computational and Graphical Statistics, vol 26, no 3, pp 513-524.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### See Also

contourmap.inla, contourmap.mc, contourmap.colors

## **Examples**

```
n <- 10
Q <- Matrix(toeplitz(c(1, -0.5, rep(0, n - 2))))
mu <- seq(-5, 5, length = n)
lp <- contourmap(mu, Q,
    n.levels = 2,
    compute = list(F = FALSE, measures = c("P1", "P2")),
    max.threads = 1
)
# Plot the contourmap
plot(lp$map)
# Display the quality measures
cat(c(lp$P1, lp$P2))</pre>
```

contourmap.colors

Define a color map for displaying contour maps.

# Description

contourmap.colors calculates suitable colours for displaying contour maps.

## Usage

```
contourmap.colors(lp, zlim, col, credible.col)
```

## **Arguments**

| lp           | $A \ contour map \ calculated \ by \ contour map, \ contour map. in \texttt{la}, \ or \ contour map. mc$                         |
|--------------|--|
| zlim         | The range that should be used (optional). The default is the range of the mean value function used when creating the contourmap. |
| col          | The colormap that the colours should be taken from.  |
| credible.col | The color that should be used for displaying the credible regions for the contour curves (optional).                             |

#### Value

A color map.

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#### Author(s)

David Bolin <davidbolin@gmail.com>

# **Examples**

```
n <- 10
Q <- Matrix(toeplitz(c(1, -0.5, rep(0, n - 2))))
map <- contourmap(
   mu = seq(-5, 5, length = n), Q, n.levels = 2,
   compute = list(F = FALSE), max.threads = 1
)
cols <- contourmap.colors(map,
   col = heat.colors(100, 1),
   credible.col = grey(0.5, 1)
)</pre>
```

contourmap.inla

Contour maps and contour map quality measures for latent Gaussian models

# Description

An interface to the contourmap function for latent Gaussian models calculated using the INLA method.

## Usage

```
contourmap.inla(
  result.inla,
  stack,
 name = NULL,
  tag = NULL,
 method = "QC",
  n.levels,
  type = c("standard", "pretty", "equalarea"),
  compute = list(F = TRUE, measures = NULL),
  alpha,
  F.limit,
  n.iter = 10000,
  verbose = FALSE,
 max.threads = 0,
  compressed = TRUE,
  seed = NULL,
  ind,
)
```

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#### **Arguments**

result.inla Result object from INLA call.

stack The stack object used in the INLA call.

name The name of the component for which to do the calculation. This argument

should only be used if a stack object is not provided, use the tag argument oth-

erwise.

The tag of the component in the stack for which to do the calculation. This ar-

gument should only be used if a stack object is provided, use the name argument

otherwise.

method Method for handeling the latent Gaussian structure. Currently only Empirical

Bayes (EB) and Quantile corrections (QC) are supported.

n. levels Number of levels in contour map.

type Type of contour map. One of:

'standard' Equidistant levels between smallest and largest value of the poste-

rior mean (default).

'pretty' Equally spaced 'round' values which cover the range of the values in

the posterior mean.

'equalarea' Levels such that different spatial regions are approximately equal

in size.

compute A list with quality indices to compute

'F': TRUE/FALSE indicating whether the contour map function should be

computed (default TRUE)

'measures': A list with the quality measures to compute ("P0", "P1", "P2") or corresponding bounds based only on the marginal probabilities ("P0-

bound", "P1-bound", "P2-bound")

alpha Maximal error probability in contour map function (default=1)

F.limit The limit value for the computation of the F function. F is set to NA for all

nodes where F<1-F.limit. Default is F.limit = alpha.

n.iter Number or iterations in the MC sampler that is used for calculating the quantities

in compute. The default value is 10000.

verbose Set to TRUE for verbose mode (optional)

max.threads Decides the number of threads the program can use. Set to 0 for using the

maximum number of threads allowed by the system (default).

compressed If INLA is run in compressed mode and a part of the linear predictor is to be

used, then only add the relevant part. Otherwise the entire linear predictor is

added internally (default TRUE).

seed Random seed (optional).

ind If only a part of a component should be used in the calculations, this argument

specifies the indices for that part (optional).

... Additional arguments to the contour map function. See the documentation for

contourmap for details.

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#### **Details**

The INLA approximation of the quantity of interest is in general a weighted sum of Gaussian distributions with different parameters. If method = 'EB' is used, then the contour map is computed for the mean of the component in the weighted sum that has parameters with the highest likelihood. If on the other hand method='QC', then the contour map is computed for the posterior mean reported by INLA. If the EB method also is used in INLA, then this reported posterior mean is equal to the mean of the component with the highest likelihood. Therefore, method='EB' is appropriate if the EB method also is used in INLA, but method='QC' should be used in general.

The n.levels contours in the contour map are are placed according to the argument type. A number of quality measures can be computed based based on the specified contour map and the distribution of the component of interest. What should be computed is specified using the compute argument. For details on these quanties, see the reference below.

#### Value

contourmap.inla returns an object of class "excurobj" with the same elements as returned by contourmap.

#### Note

This function requires the INLA package, which is not a CRAN package. See https://www.r-inla.org/download-install for easy installation instructions.

## Author(s)

David Bolin <davidbolin@gmail.com>

## References

Bolin, D. and Lindgren, F. (2017) *Quantifying the uncertainty of contour maps*, Journal of Computational and Graphical Statistics, 26:3, 513-524.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

## See Also

contourmap, contourmap.mc, contourmap.colors

```
## Not run:
if (require.nowarnings("INLA")) {
    # Generate mesh and SPDE model
    n.lattice <- 10 # increase for more interesting, but slower, examples
    x <- seq(from = 0, to = 10, length.out = n.lattice)
    lattice <- fmesher::fm_lattice_2d(x = x, y = x)
    mesh <- fmesher::fm_rcdt_2d_inla(lattice = lattice, extend = FALSE, refine = FALSE)
    spde <- inla.spde2.matern(mesh, alpha = 2)</pre>
```

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```
# Generate an artificial sample
 sigma2.e <- 0.01
 n.obs <- 100
 obs.loc <- cbind(</pre>
    runif(n.obs) * diff(range(x)) + min(x),
    runif(n.obs) * diff(range(x)) + min(x)
 Q \leftarrow inla.spde2.precision(spde, theta = c(log(sqrt(0.5)), log(sqrt(1))))
 x \leftarrow inla.qsample(Q = Q)
 A <- fmesher::fm_basis(mesh = mesh, loc = obs.loc)
 Y <- as.vector(A %*% x + rnorm(n.obs) * sqrt(sigma2.e))
 ## Estimate the parameters using INLA
 mesh.index <- inla.spde.make.index(name = "field", n.spde = spde$n.spde)</pre>
 ef <- list(c(mesh.index, list(Intercept = 1)))</pre>
 s.obs <- inla.stack(data = list(y = Y), A = list(A), effects = ef, tag = "obs")</pre>
 s.pre <- inla.stack(data = list(y = NA), A = list(1), effects = ef, tag = "pred")</pre>
 stack <- inla.stack(s.obs, s.pre)</pre>
 formula \leftarrow y \sim -1 + Intercept + f(field, model = spde)
 result <- inla(
    formula = formula, family = "normal", data = inla.stack.data(stack),
    control.predictor = list(
      A = inla.stack.A(stack),
      compute = TRUE
    control.compute = list(
      config = TRUE,
      return.marginals.predictor = TRUE
   ),
   num.threads = 1
 )
 ## Calculate contour map with two levels
 map <- contourmap.inla(result,</pre>
    stack = stack, tag = "pred",
   n.levels = 2, alpha = 0.1, F.limit = 0.1,
   max.threads = 1
 ## Plot the results
 cols <- contourmap.colors(map,</pre>
    col = heat.colors(100, 1),
    credible.col = grey(0.5, 1)
 image(matrix(map$M[mesh$idx$lattice], n.lattice, n.lattice), col = cols)
## End(Not run)
```

}

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contourmap.mc

Contour maps and contour map quality measures using Monte Carlo samples

#### **Description**

contourmap.mc is used for calculating contour maps and quality measures for contour maps based on Monte Carlo samples of a model.

#### Usage

```
contourmap.mc(
  samples,
  n.levels,
  ind,
  levels,
  type = c("standard", "equalarea", "P0-optimal", "P1-optimal", "P2-optimal"),
  compute = list(F = TRUE, measures = NULL),
  alpha,
  verbose = FALSE
)
```

#### **Arguments**

samples Matrix with model Monte Carlo samples. Each column contains a sample of the

model.

n.levels Number of levels in contour map.

ind Indices of the nodes that should be analyzed (optional).

levels Levels to use in contour map. type Type of contour map. One of:

'standard' Equidistant levels between smallest and largest value of the posterior mean (default).

'pretty' Equally spaced 'round' values which cover the range of the values in the posterior mean.

'equalarea' Levels such that different spatial regions are approximately equal in size.

**'P0-optimal'** Levels chosen to maximize the P0 measure.

'P1-optimal' Levels chosen to maximize the P1 measure.

'P2-optimal' Levels chosen to maximize the P2 measure.

compute A list with quality indices to compute

**'F':** TRUE/FALSE indicating whether the contour map function should be computed (default TRUE).

'measures': A list with the quality measures to compute ("P0", "P1", "P2") or corresponding bounds based only on the marginal probabilities ("P0-bound", "P1-bound", "P2-bound").

alpha Maximal error probability in contour map function (default=0.1).

verbose Set to TRUE for verbose mode (optional).

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#### **Details**

The contour map is computed for the empirical mean of the samples. See contourmap and contourmap.inla for further details.

#### Value

contourmap returns an object of class "excurobj" with the following elements

| u               | Contour levels used in the contour map.   |
|-----------------|---|
| n.levels        | The number of contours used.  |
| u.e             | The values associated with the level sets G_k.  |
| G               | A vector which shows which of the level sets G_k each node belongs to.  |
| map             | Representation of the contour map with map[i]=u.e[k] if i is in G_k.  |
| F               | The contour map function (if computed).   |
| М               | Contour avoiding sets (if F is computed). $M=-1$ for all non-significant nodes and $M=k$ for nodes that belong to $M_k$ . |
| P0/P1/P2        | Calculated quality measures (if computed).  |
| P0bound/P1bound | I/P2bound   |
|                 | Calculated upper bounds quality measures (if computed).   |
| meta            | A list containing various information about the calculation.  |

## Author(s)

David Bolin <davidbolin@gmail.com>

#### References

Bolin, D. and Lindgren, F. (2017) *Quantifying the uncertainty of contour maps*, Journal of Computational and Graphical Statistics, 26:3, 513-524.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, 86(5), 1–20.

#### See Also

```
contourmap.colors
```

```
n <- 100
Q <- Matrix(toeplitz(c(1, -0.5, rep(0, n - 2))))
mu <- seq(-5, 5, length = n)
## Sample the model 100 times (increase for better estimate)
X <- mu + solve(chol(Q), matrix(rnorm(n = n * 100), nrow = n, ncol = 100))

lp <- contourmap.mc(X, n.levels = 2, compute = list(F = FALSE, measures = c("P1", "P2")))
# plot contourmap</pre>
```

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```
plot(lp$map)
# display quality measures
c(lp$P1, lp$P2)
```

excursions

Excursion Sets and Contour Credibility Regions for Random Fields

## **Description**

excursions is one of the main functions in the package with the same name. For an introduction to the package, see excursions-package. The function is used for calculating excursion sets, contour credible regions, and contour avoiding sets for latent Gaussian models. Details on the function and the package are given in the sections below.

## Usage

```
excursions(
  alpha,
  u,
 mu,
  Q,
  type,
 n.iter = 10000,
 Q.chol,
 F.limit,
  vars,
  rho,
  reo,
 method = "EB",
  ind,
 max.size,
 verbose = 0,
 max.threads = 0,
  seed
)
```

## **Arguments**

alpha Error probability for the excursion set.

u Excursion or contour level.

mu Expectation vector.

Q Precision matrix.

type Type of region:

'>' positive excursion region

'<' negative excursion region

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|             | '!=' contour avoiding region   |
|-------------|--|
|             | '=' contour credibility region   |
| n.iter      | Number or iterations in the MC sampler that is used for approximating probabilities. The default value is 10000.                     |
| Q.chol      | The Cholesky factor of the precision matrix (optional).  |
| F.limit     | The limit value for the computation of the F function. F is set to NA for all nodes where F<1-F.limit. Default is F.limit = alpha.   |
| vars        | Precomputed marginal variances (optional).   |
| rho         | Marginal excursion probabilities (optional). For contour regions, provide $P(X>u)$ .   |
| reo         | Reordering (optional).   |
| method      | Method for handeling the latent Gaussian structure:  |
|             | 'EB' Empirical Bayes (default)   |
|             | 'QC' Quantile correction, rho must be provided if QC is used.  |
| ind         | Indices of the nodes that should be analysed (optional).   |
| max.size    | Maximum number of nodes to include in the set of interest (optional).  |
| verbose     | Set to TRUE for verbose mode (optional).   |
| max.threads | Decides the number of threads the program can use. Set to 0 for using the maximum number of threads allowed by the system (default). |
| seed        | Random seed (optional).  |

## **Details**

The estimation of the region is done using sequential importance sampling with n.iter samples. The procedure requires computing the marginal variances of the field, which should be supplied if available. If not, they are computed using the Cholesky factor of the precision matrix. The cost of this step can therefore be reduced by supplying the Cholesky factor if it is available.

The latent structure in the latent Gaussian model can be handled in several different ways. The default strategy is the EB method, which is exact for problems with Gaussian posterior distributions. For problems with non-Gaussian posteriors, the QC method can be used for improved results. In order to use the QC method, the true marginal excursion probabilities must be supplied using the argument rho. Other more complicated methods for handling non-Gaussian posteriors must be implemented manually unless INLA is used to fit the model. If the model is fitted using INLA, the method excursions.inla can be used. See the Package section for further details about the different options.

## Value

excursions returns an object of class "excurobj" with the following elements

| Е | Excursion set, contour credible region, or contour avoiding set |
|---|---|
| G | Contour map set. $G = 1$ for all nodes where the $mu > u$ .     |

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| М    | Contour avoiding set. $M=-1$ for all non-significant nodes. $M=0$ for nodes where the process is significantly below u and $M=1$ for all nodes where the field is significantly above u. Which values that should be present depends on what type of set that is calculated. |
|------|--|
| F    | The excursion function corresponding to the set ${\sf E}$ calculated or values up to ${\sf F.limit}$   |
| rho  | Marginal excursion probabilities   |
| mean | The mean mu.   |
| vars | Marginal variances.  |
| meta | A list containing various information about the calculation.   |

#### Author(s)

David Bolin <davidbolin@gmail.com> and Finn Lindgren <finn.lindgren@gmail.com>

#### References

Bolin, D. and Lindgren, F. (2015) *Excursion and contour uncertainty regions for latent Gaussian models*, JRSS-series B, vol 77, no 1, pp 85-106.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### See Also

excursions-package, excursions.inla, excursions.mc

```
## Create a tridiagonal precision matrix
n <- 21
Q.x <- sparseMatrix(
    i = c(1:n, 2:n), j = c(1:n, 1:(n - 1)), x = c(rep(1, n), rep(-0.1, n - 1)),
    dims = c(n, n), symmetric = TRUE
)
## Set the mean value function
mu.x <- seq(-5, 5, length = n)

## calculate the level 0 positive excursion function
res.x <- excursions(
    alpha = 1, u = 0, mu = mu.x, Q = Q.x,
    type = ">", verbose = 1, max.threads = 2
)

## Plot the excursion function and the marginal excursion probabilities
plot(res.x$F,
    type = "1",
    main = "Excursion function (black) and marginal probabilites (red)"
)
lines(res.x$rho, col = 2)
```

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| excursions.inla | Excursion sets and contour credible regions for latent Gaussian models |
|-----------------|--|
|                 | els  |

# Description

Excursion sets and contour credible regions for latent Gaussian models calculated using the INLA method.

# Usage

```
excursions.inla(
  result.inla,
  stack,
  name = NULL,
  tag = NULL,
  ind = NULL,
  method,
  alpha = 1,
  F.limit,
  u,
  u.link = FALSE,
  type,
  n.iter = 10000,
  verbose = 0,
  \max.threads = 0,
  compressed = TRUE,
  seed = NULL
)
```

# Arguments

| result.inla | Result object from INLA call.  |
|-------------|--|
| stack       | The stack object used in the INLA call.  |
| name        | The name of the component for which to do the calculation. This argument should only be used if a stack object is not provided, use the tag argument otherwise.          |
| tag         | The tag of the component in the stack for which to do the calculation. This argument should only be used if a stack object is provided, use the name argument otherwise. |
| ind         | If only a part of a component should be used in the calculations, this argument specifies the indices for that part.   |
| method      | Method for handeling the latent Gaussian structure:  |
|             | 'EB' Empirical Bayes   |
|             | 'QC' Quantile correction   |

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'NI' Numerical integration

'NIQC' Numerical integration with quantile correction 'iNIQC' Improved integration with quantile correction

alpha Error probability for the excursion set of interest. The default value is 1.

F.limit Error probability for when to stop the calculation of the excursion function. The

default value is alpha, and the value cannot be smaller than alpha. A smaller

value of F. limit results in a smaller computation time.

u Excursion or contour level.

u.link If u.link is TRUE, u is assumed to be in the scale of the data and is then trans-

formed to the scale of the linear predictor (default FALSE).

type Type of region:

'>' positive excursions'<' negative excursions</li>

'!=' contour avoiding function'=' contour credibility function

n.iter Number or iterations in the MC sampler that is used for approximating proba-

bilities. The default value is 10000.

verbose Set to TRUE for verbose mode (optional).

max.threads Decides the number of threads the program can use. Set to 0 for using the

maximum number of threads allowed by the system (default).

compressed If INLA is run in compressed mode and a part of the linear predictor is to be

used, then only add the relevant part. Otherwise the entire linear predictor is

added internally (default TRUE).

seed Random seed (optional).

# Details

The different methods for handling the latent Gaussian structure are listed in order of accuracy and computational cost. The EB method is the simplest and is based on a Gaussian approximation of the posterior of the quantity of interest. The QC method uses the same Gaussian approximation but improves the accuracy by modifying the limits in the integrals that are computed in order to find the region. The other three methods are intended for Bayesian models where the posterior distribution for the quantity of interest is obtained by integrating over the parameters in the model. The NI method approximates this integration in the same way as is done in INLA, and the NIQC and iNIQC methods combine this apprximation with the QC method for improved accuracy.

If the main purpose of the analysis is to construct excursion or contour sets for low values of alpha, we recommend using QC for problems with Gaussian likelihoods and NIQC for problems with non-Gaussian likelihoods. The reason for this is that the more accurate methods also have higher computational costs.

#### Value

excursions.inla returns an object of class "excurobj" with the following elements

E Excursion set, contour credible region, or contour avoiding set

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| F    | The excursion function corresponding to the set E calculated for values up to F.limit  |
|------|--|
| G    | Contour map set. $G = 1$ for all nodes where the $mu > u$ .  |
| М    | Contour avoiding set. $M=-1$ for all non-significant nodes. $M=0$ for nodes where the process is significantly below u and $M=1$ for all nodes where the field is significantly above u. Which values that should be present depends on what type of set that is calculated. |
| rho  | Marginal excursion probabilities   |
| mean | Posterior mean   |
| vars | Marginal variances   |
| meta | A list containing various information about the calculation.   |

# Note

This function requires the INLA package, which is not a CRAN package. See https://www.r-inla.org/download-install for easy installation instructions.

#### Author(s)

David Bolin <davidbolin@gmail.com> and Finn Lindgren <finn.lindgren@gmail.com>

#### References

Bolin, D. and Lindgren, F. (2015) *Excursion and contour uncertainty regions for latent Gaussian models*, JRSS-series B, vol 77, no 1, pp 85-106.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### See Also

excursions, excursions.mc

```
## In this example, we calculate the excursion function
## for a partially observed AR process.
## Not run:
if (require.nowarnings("INLA")) {
    ## Sample the process:
    rho <- 0.9
    tau <- 15
    tau.e <- 1
    n <- 100
    x <- 1:n
    mu <- 10 * ((x < n / 2) * (x - n / 2) + (x >= n / 2) * (n / 2 - x) + n / 4) / n
Q <- tau * sparseMatrix(
    i = c(1:n, 2:n), j = c(1:n, 1:(n - 1)),
    x = c(1, rep(1 + rho^2, n - 2), 1, rep(-rho, n - 1)),
    dims = c(n, n), symmetric = TRUE</pre>
```

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```
X <- mu + solve(chol(Q), rnorm(n))</pre>
 ## measure the sampled process at n.obs random locations
 ## under Gaussian measurement noise.
 n.obs <- 50
 obs.loc <- sample(1:n, n.obs)</pre>
 A <- sparseMatrix(
    i = 1:n.obs, j = obs.loc, x = rep(1, n.obs),
   dims = c(n.obs, n)
 Y <- as.vector(A %*% X + rnorm(n.obs) / sqrt(tau.e))
 ## Estimate the parameters using INLA
 ef <- list(c(list(ar = x), list(cov = mu)))</pre>
 s.obs <- inla.stack(data = list(y = Y), A = list(A), effects = ef, tag = "obs")</pre>
 s.pre <- inla.stack(data = list(y = NA), A = list(1), effects = ef, tag = "pred")</pre>
 stack <- inla.stack(s.obs, s.pre)</pre>
 formula \leftarrow y \sim -1 + cov + f(ar, model = "ar1")
 result <- inla(
    formula = formula, family = "normal", data = inla.stack.data(stack),
    control.predictor = list(A = inla.stack.A(stack), compute = TRUE),
   control.compute = list(
      config = TRUE,
      return.marginals.predictor = TRUE
   )
 )
 \#\# calculate the level 0 positive excursion function
 res.qc <- excursions.inla(result,</pre>
   stack = stack, tag = "pred", alpha = 0.99, u = 0,
   method = "QC", type = ">", max.threads = 2
 ## plot the excursion function and marginal probabilities
 plot(res.qc$rho,
    type = "1",
    main = "marginal probabilities (black) and excursion function (red)"
 lines(res.qc\$F, col = 2)
}
## End(Not run)
```

Excursion sets and contour credible regions using Monte Carlo samples

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## **Description**

excursions.mc is used for calculating excursion sets, contour credible regions, and contour avoiding sets based on Monte Carlo samples of models.

#### **Usage**

```
excursions.mc(
  samples,
  alpha,
  u,
  type,
  rho,
  reo,
  ind,
  max.size,
  verbose = FALSE
)
```

## **Arguments**

| samples | Matrix with model Monte Carlo samples. E | Each column contains a sample of the |
|---------|--|--------------------------------------|
|---------|--|--------------------------------------|

model.

alpha Error probability for the excursion set.

u Excursion or contour level.

type Type of region:

'>' positive excursions'<' negative excursions</li>

'!=' contour avoiding function'=' contour credibility function

rho Marginal excursion probabilities (optional). For contour regions, provide P(X >

u).

reo Reordering (optional).

ind Indices of the nodes that should be analysed (optional).

max.size Maximum number of nodes to include in the set of interest (optional).

verbose Set to TRUE for verbose mode (optional).

#### Value

М

excursions.mc returns an object of class "excurobj" with the following elements

E Excursion set, contour credible region, or contour avoiding set.

G Contour map set. G = 1 for all nodes where the mu > u.

Contour avoiding set. M=-1 for all non-significant nodes. M=0 for nodes where the process is significantly below u and M=1 for all nodes where the field is significantly above u. Which values that should be present depends on

what type of set that is calculated.

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| F    | The excursion function corresponding to the set ${\tt E}$ calculated for values up to ${\tt F.limit}$ |
|------|---|
| rho  | Marginal excursion probabilities  |
| mean | The mean mu.  |
| vars | Marginal variances.   |
| meta | A list containing various information about the calculation.  |

#### Author(s)

David Bolin <davidbolin@gmail.com> and Finn Lindgren <finn.lindgren@gmail.com>

#### References

Bolin, D. and Lindgren, F. (2015) *Excursion and contour uncertainty regions for latent Gaussian models*, JRSS-series B, vol 77, no 1, pp 85-106.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### See Also

```
excursions, excursions.inla
```

## **Examples**

```
## Create mean and a tridiagonal precision matrix n <-101 mu.x <-seq(-5, 5, length = n) Q.x <- Matrix(toeplitz(c(1, -0.1, rep(0, n - 2)))) ## Sample the model 100 times (increase for better estimate) X <- mu.x + solve(chol(Q.x), matrix(rnorm(n = n * 1000), nrow = n, ncol = 1000)) ## calculate the positive excursion function res.x <- excursions.mc(X, alpha = 0.05, type = ">", u = 0) ## Plot the excursion function and the marginal excursion probabilities plot(res.x$F, type = "1", main = "Excursion function (black) and marginal probabilities (red)" ) lines(res.x$rho, col = 2)
```

excursions.variances Calculate variances from a sparse precision matrix

## **Description**

excursions.variances calculates the diagonal of the inverse of a sparse symmetric positive definite matrix Q.

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#### Usage

```
excursions.variances(L, Q, max.threads = 0)
```

## **Arguments**

L Cholesky factor of precision matrix.

O Precision matrix.

max.threads Decides the number of threads the program can use. Set to 0 for using the

maximum number of threads allowed by the system (default).

#### **Details**

The method for calculating the diagonal requires the Cholesky factor, L, of Q, which should be supplied if available. If Q is provided, the cholesky factor is calculated and the variances are then returned in the same ordering as Q. If L is provided, the variances are returned in the same ordering as L, even if L@invpivot exists.

#### Value

A vector with the variances.

#### Author(s)

David Bolin <davidbolin@gmail.com>

## **Examples**

```
## Create a tridiagonal precision matrix n \leftarrow 21 Q \leftarrow Matrix(toeplitz(c(1, -0.1, rep(0, n - 2)))) v2 \leftarrow excursions.variances(Q = Q, max.threads = 2) ## var2 should be the same as: v1 \leftarrow diag(solve(Q))
```

exc\_safe\_inla

Load INLA safely for examples and tests

## **Description**

Loads the INLA package with 'requireNamespace("INLA", quietly = TRUE)', and optionally checks and sets the multicore 'num.threads' INLA option.

## Usage

```
exc_safe_inla(multicore = NULL, quietly = FALSE)
```

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## Arguments

multicore logical; if 'TRUE', multiple cores are allowed, and the INLA 'num.threads'

option is not checked or altered. If 'FALSE', forces 'num.threads="1:1"'. Default: NULL, checks if running in testthat or non-interactively, in which case

sets 'multicore=FALSE', otherwise 'TRUE'.

quietly logical; if 'TRUE', prints diagnostic messages. Default: FALSE.

#### Value

logical; 'TRUE' if INLA was loaded safely, otherwise FALSE

## **Examples**

```
## Not run:
if (exc_safe_inla()) {
    # Run inla dependent calculations
}
## End(Not run)
```

gaussint

Sequential estimation of Gaussian integrals

## **Description**

gaussint is used for calculating n-dimensional Gaussian integrals

$$\int_a^b \frac{|Q|^{1/2}}{(2\pi)^{n/2}} \exp(-\frac{1}{2}(x-\mu)^T Q(x-\mu)) dx$$

A limit value lim can be used to stop the integration if the sequential estimate goes below the limit, which can result in substantial computational savings in cases when one only is interested in testing if the integral is above the limit value. The integral is calculated sequentially, and estimates for all subintegrals are also returned.

## Usage

```
gaussint(
   mu,
   Q.chol,
   Q,
   a,
   b,
   lim = 0,
   n.iter = 10000,
   ind,
```

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```
use.reordering = c("natural", "sparsity", "limits"),
max.size,
max.threads = 0,
seed
)
```

#### **Arguments**

| mu             | Expectation vector for the Gaussian distribution.   |
|----------------|---|
| Q.chol         | The Cholesky factor of the precision matrix (optional).   |
| Q              | Precision matrix for the Gaussian distribution. If Q is supplied but not Q.chol, the cholesky factor is computed before integrating.  |
| a              | Lower limit in integral.  |
| b              | Upper limit in integral.  |
| lim            | If this argument is used, the integration is stopped and $0$ is returned if the estimated value goes below $\lim$ .                   |
| n.iter         | Number or iterations in the MC sampler that is used for approximating probabilities. The default value is 10000.                      |
| ind            | Indices of the nodes that should be analyzed (optional).  |
| use.reordering | Determines what reordering to use:  |
|                | "natural" No reordering is performed.   |
|                | <b>"sparsity"</b> Reorder for sparsity in the cholesky factor (MMD reordering is used).   |
|                | <b>''limits''</b> Reorder by moving all nodes with a=-Inf and b=Inf first and then reordering for sparsity (CAMD reordering is used). |
| max.size       | The largest number of sub-integrals to compute. Default is the total dimension of the distribution.                                   |
| max.threads    | Decides the number of threads the program can use. Set to 0 for using the maximum number of threads allowed by the system (default).  |
| seed           | The random seed to use (optional).  |

#### **Details**

The function uses sequential importance sampling to estimate the Gaussian integral, and returns all computed sub-integrals. This means that if, for example, the function is used to compute P(x>0) for an n-dimensional Gaussian variable x, then all integrals  $P(x_1>0,\ldots,x_i>0)$  for  $i=1,\ldots,n$  are computed.

If one is only interested in whether  $P(x>0)>\alpha$  or not, then one can stop the integration as soon as  $P(x_1>0,\ldots,x_i>0)<\alpha$ . This can save a lot of computation time if  $P(x_1>0,\ldots,x_i>0)<\alpha$  for i much smaller than n. This limit value is specified by the lim argument.

Which reordering to use depends on what the purpose of the calculation is and what the integration limits are. However, in general the limits reordering is typically most appropriate since this combines sparisty (which improves accuracy and reduces computational cost) with automatic handling of dimensions with limits a=-Inf and b=Inf, which do not affect the probability but affect the computation time if they are not handled separately.

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#### Value

A list with elements

P Value of the integral.

E Estimated error of the P estimate.

Pv A vector with the estimates of all sub-integrals.

Ev A vector with the estimated errors of the Pv estimates.

#### Author(s)

David Bolin <davidbolin@gmail.com>

#### References

Bolin, D. and Lindgren, F. (2015) Excursion and contour uncertainty regions for latent Gaussian models, JRSS-series B, vol 77, no 1, pp 85-106.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### **Examples**

```
## Create mean and a tridiagonal precision matrix n <-11 mu.x <- seq(-5, 5, length = n) Q.x <- Matrix(toeplitz(c(1, -0.1, rep(0, n - 2)))) ## Calculate the probability that the variable is between mu-3 and mu+3 prob <- gaussint(mu = mu.x, Q = Q.x, a = mu.x - 3, b = mu.x + 3, max.threads = 2) prob$P
```

require.nowarnings

Warnings free loading of add-on packages

## **Description**

Turn off all warnings for require(), to allow clean completion of examples that require unavailable Suggested packages.

## Usage

```
require.nowarnings(package, lib.loc = NULL, character.only = FALSE)
```

## **Arguments**

package The name of a package, given as a character string.

lib.loc a character vector describing the location of R library trees to search through, or

NULL. The default value of NULL corresponds to all libraries currently known to

.libPaths(). Non-existent library trees are silently ignored.

character.only a logical indicating whether package can be assumed to be a character string.

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#### **Details**

require(package) acts the same as require(package, quietly = TRUE) but with warnings turned off. In particular, no warning or error is given if the package is unavailable. Most cases should use requireNamespace(package, quietly = TRUE) instead, which doesn't produce warnings.

#### Value

require.nowarnings returns (invisibly) TRUE if it succeeds, otherwise FALSE

#### See Also

```
require
```

## **Examples**

```
## This should produce no output:
if (require.nowarnings(nonexistent)) {
  message("Package loaded successfully")
}
```

simconf

Simultaneous confidence regions for Gaussian models

## Description

simconf is used for calculating simultaneous confidence regions for Gaussian models x. The function returns upper and lower bounds a and b such that  $P(a < x < b) = 1 - \alpha$ .

# Usage

```
simconf(
  alpha,
  mu,
  Q,
  n.iter = 10000,
  Q.chol,
  vars,
  ind = NULL,
  verbose = 0,
  max.threads = 0,
  seed = NULL
)
```

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#### **Arguments**

alpha Error probability for the region.

mu Expectation vector for the Gaussian distribution.

Q Precision matrix for the Gaussian distribution.

n.iter Number or iterations in the MC sampler that is used for approximating proba-

bilities. The default value is 10000.

Q. chol The Cholesky factor of the precision matrix (optional).

vars Precomputed marginal variances (optional).

ind Indices of the nodes that should be analyzed (optional).

verbose Set to TRUE for verbose mode (optional).

max.threads Decides the number of threads the program can use. Set to 0 for using the

maximum number of threads allowed by the system (default).

seed Random seed (optional).

#### **Details**

The pointwise confidence bands are based on the marginal quantiles, meaning that a marignal is a vector where the ith element equals  $\mu_i + q_{\alpha,i}$  and b marginal is a vector where the ith element equals  $\mu_i + q_{1-\alpha,i}$ , where  $\mu_i$  is the expected value of the  $x_i$  and  $q_{\alpha,i}$  is the  $\alpha$ -quantile of  $x_i - \mu_i$ .

The simultaneous confidence band is defined by the lower limit vector a and the upper limit vector b, where  $a_i = \mu_i + cq_{\alpha}$  and  $b_i = \mu_i + cq_{1-\alpha}$ , where c is a constant computed such that  $P(a < x < b) = 1 - \alpha$ .

#### Value

An object of class "excurobj" with elements

a The lower bound.b The upper bound.

a.marginal The lower bound for pointwise confidence bands.b.marginal The upper bound for pointwise confidence bands.

#### Author(s)

David Bolin <davidbolin@gmail.com> and Finn Lindgren <finn.lindgren@gmail.com>

#### References

Bolin et al. (2015) *Statistical prediction of global sea level from global temperature*, Statistica Sinica, vol 25, pp 351-367.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### See Also

simconf.inla, simconf.mc, simconf.mixture

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#### **Examples**

```
## Create mean and a tridiagonal precision matrix
n <- 11
mu.x <- seq(-5, 5, length = n)
Q.x <- Matrix(toeplitz(c(1, -0.1, rep(0, n - 2))))
## calculate the confidence region
conf <- simconf(0.05, mu.x, Q.x, max.threads = 2)
## Plot the region
plot(mu.x,
   type = "1", ylim = c(-10, 10),
   main = "Mean (black) and confidence region (red)"
)
lines(conf$a, col = 2)
lines(conf$b, col = 2)</pre>
```

simconf.inla

Simultaneous confidence regions for latent Gaussian models

## **Description**

simconf.inla is used for calculating simultaneous confidence regions for latent Gaussian models estimated using INLA.

#### Usage

```
simconf.inla(
  result.inla,
  stack,
  name = NULL,
  tag = NULL,
  ind = NULL,
  alpha,
  method = "NI",
  n.iter = 10000,
  verbose = 0,
  link = FALSE,
  max.threads = 0,
  seed = NULL,
  inla.sample = TRUE
)
```

#### **Arguments**

result.inla Result object from INLA call.

stack The stack object used in the INLA call.

name The name of the component for which to do the calculation. This argument

should only be used if a stack object is not provided, use the tag argument oth-

erwise.

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The tag of the component in the stack for which to do the calculation. This ar-

gument should only be used if a stack object is provided, use the name argument

otherwise.

ind If only a part of a component should be used in the calculations, this argument

specifies the indices for that part.

alpha Error probability for the region.

method Method for handling the latent Gaussian structure:

'EB' Empirical Bayes (Gaussian approximation of posterior).

'NI' Numerical integration (Calculation based on the Gaussian mixture ap-

proximation of the posterior, as calculated by INLA).

n.iter Number or iterations in the MC sampler that is used for approximating proba-

bilities. The default value is 10000.

verbose Set to TRUE for verbose mode (optional).

link Transform output to the scale of the data using the link function as defined in

the model estimated with INLA (default FALSE).

max.threads Decides the number of threads the program can use. Set to 0 for using the

maximum number of threads allowed by the system (default).

seed Random seed (optional).

inla.sample Set to TRUE if inla.posterior.sample should be used for the MC integration.

#### **Details**

See simconf for details.

## Value

An object of class "excurobj" with elements

a The lower bound.b The upper bound.

a.marginal The lower bound for pointwise confidence bands.b.marginal The upper bound for pointwise confidence bands.

#### Note

This function requires the INLA package, which is not a CRAN package. See https://www.r-inla.org/download-install for easy installation instructions.

#### Author(s)

David Bolin <davidbolin@gmail.com>

#### References

Bolin et al. (2015) Statistical prediction of global sea level from global temperature, Statistica Sinica, vol 25, pp 351-367.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

simconf.mc 33

#### See Also

```
simconf, simconf.mc, simconf.mixture
```

#### **Examples**

```
## Not run:
if (require.nowarnings("INLA")) {
 n <- 10
 x \leftarrow seq(0, 6, length.out = n)
 y <- \sin(x) + rnorm(n)
 mu <- 1:n
 result <- inla(y ~ 1 + f(mu, model = "rw2"),
    data = list(y = y, mu = mu), verbose = FALSE,
   control.compute = list(
      config = TRUE,
      return.marginals.predictor = TRUE
   ),
    num.threads = "1:1"
 res <- simconf.inla(result, name = "mu", alpha = 0.05, max.threads = 1)</pre>
 plot(resultsummary.randomsmusmean, ylim = c(-2, 2))
 lines(res$a)
 lines(res$b)
 lines(res$a.marginal, col = "2")
 lines(res$b.marginal, col = "2")
}
## End(Not run)
```

simconf.mc

Simultaneous confidence regions using Monte Carlo samples

## **Description**

simconf.mc is used for calculating simultaneous confidence regions based on Monte Carlo samples. The function returns upper and lower bounds a and b such that  $P(a < x < b) = 1 - \alpha$ .

#### Usage

```
simconf.mc(samples, alpha, ind, verbose = FALSE)
```

#### **Arguments**

samples Matrix with model Monte Carlo samples. Each column contains a sample of the model.

alpha Error probability for the region.

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ind Indices of the nodes that should be analyzed (optional).

verbose Set to TRUE for verbose mode (optional).

#### **Details**

See simconf for details.

#### Value

An object of class "excurobj" with elements

a The lower bound.b The upper bound.

a.marginal The lower bound for pointwise confidence bands.b.marginal The upper bound for pointwise confidence bands.

## Author(s)

David Bolin <davidbolin@gmail.com>

# See Also

```
simconf, simconf.inla
```

```
## Create mean and a tridiagonal precision matrix
n <- 11
mu.x <- seq(-5, 5, length = n)
Q.x <- Matrix(toeplitz(c(1, -0.1, rep(0, n - 2))))
## Sample the model 100 times (increase for better estimate)
X <- mu.x + solve(chol(Q.x), matrix(rnorm(n = n * 100), nrow = n, ncol = 100))
## calculate the confidence region
conf <- simconf.mc(X, 0.2)
## Plot the region
plot(mu.x,
    type = "l", ylim = c(-10, 10),
    main = "Mean (black) and confidence region (red)"
)
lines(conf$a, col = 2)
lines(conf$b, col = 2)</pre>
```

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simconf.mixture

Simultaneous confidence regions for Gaussian mixture models

# Description

simconf.mixture is used for calculating simultaneous confidence regions for Gaussian mixture models. The distribution for the process  $\boldsymbol{x}$  is assumed to be

$$\pi(x) = \sum_{k=1}^{K} w_k N(\mu_k, Q_k^{-1}).$$

The function returns upper and lower bounds a and b such that  $P(a < x < b) = 1 - \alpha$ .

# Usage

```
simconf.mixture(
   alpha,
   mu,
   Q,
   w,
   ind,
   n.iter = 10000,
   vars,
   verbose = 0,
   max.threads = 0,
   seed = NULL,
   mix.samp = TRUE
)
```

# Arguments

| alpha       | Error probability for the region.  |
|-------------|--|
| mu          | A list with the k expectation vectors $\mu_k$ .  |
| Q           | A list with the k precision matrices $Q_k$ .   |
| W           | A vector with the weights for each class in the mixture.   |
| ind         | Indices of the nodes that should be analyzed (optional).   |
| n.iter      | Number or iterations in the MC sampler that is used for approximating probabilities. The default value is 10000.                     |
| vars        | A list with precomputed marginal variances for each class (optional).  |
| verbose     | Set to TRUE for verbose mode (optional).   |
| max.threads | Decides the number of threads the program can use. Set to 0 for using the maximum number of threads allowed by the system (default). |
| seed        | Random seed (optional).  |
| mix.samp    | If TRUE, the MC integration is done by directly sampling the mixture, otherwise sequential integration is used.                      |

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#### **Details**

See simconf for details.

#### Value

An object of class "excurobj" with elements

a The lower bound.
b The upper bound.

a.marginal The lower bound for pointwise confidence bands.b.marginal The upper bound for pointwise confidence bands.

#### Author(s)

David Bolin <davidbolin@gmail.com>

#### References

Bolin et al. (2015) *Statistical prediction of global sea level from global temperature*, Statistica Sinica, vol 25, pp 351-367.

Bolin, D. and Lindgren, F. (2018), *Calculating Probabilistic Excursion Sets and Related Quantities Using excursions*, Journal of Statistical Software, vol 86, no 1, pp 1-20.

#### See Also

```
simconf, simconf.inla, simconf.mc
```

```
n <- 11
K <- 3
mu <- Q <- list()
for (k in 1:K) {
    mu[[k]] <- k * 0.1 + seq(-5, 5, length = n)
    Q[[k]] <- Matrix(toeplitz(c(1, -0.1, rep(0, n - 2))))
}
## calculate the confidence region
conf <- simconf.mixture(0.05, mu, Q, w = rep(1 / 3, 3), max.threads = 2)
## Plot the region
plot(mu[[1]], type = "1")
lines(mu[[2]])
lines(mu[[3]])
lines(conf$a, col = 2)
lines(conf$b, col = 2)</pre>
```

submesh.grid 37

submesh.grid

Extract a part of a grid

# Description

Extracts a part of a grid.

## Usage

```
submesh.grid(z, grid = NULL)
```

## Arguments

z A matrix with values indicating which nodes that should be present in the sub-

mesh.

grid A list with locations and dimensions of the grid.

#### Value

An inla.mesh object.

#### Author(s)

Finn Lindgren <finn.lindgren@gmail.com>

```
## Not run:
if (require("fmesher")) {
 nxy <- 40
 x \leftarrow seq(from = 0, to = 4, length.out = nxy)
 lattice <- fm_lattice_2d(x = x, y = x)
 mesh <- fm_rcdt_2d_inla(lattice = lattice, extend = FALSE, refine = FALSE)</pre>
 # extract a part of the mesh inside a circle
 xy.in <- rowSums((mesh$loc[, 1:2] - 2)^2) < 1
 submesh <- submesh.grid(</pre>
   matrix(xy.in, nxy, nxy),
    list(loc = mesh\$loc, dim = c(nxy, nxy))
 plot(mesh$loc[, 1:2])
 lines(2 + cos(seq(0, 2 * pi, length.out = 100)), 2 + <math>sin(seq(0, 2 * pi, length.out = 100)))
 plot(submesh, add = TRUE)
 points(mesh$loc[xy.in, 1:2], col = "2")
}
## End(Not run)
```

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submesh.mesh

Extract a part of a mesh

## **Description**

Extracts a part of a mesh

## Usage

```
submesh.mesh(z, mesh)
```

# Arguments

z A matrix with values indicating which nodes that should be present in the sub-

mesh.

mesh An fm\_mesh\_2d object.

#### Value

An fm\_mesh\_2d object.

## Author(s)

Finn Lindgren <finn.lindgren@gmail.com>

```
## Not run:
if (require(fmesher)) {
    nxy <- 30
    x <- seq(from = 0, to = 4, length.out = nxy)
    lattice <- fm_lattice_2d(x = x, y = x)
    mesh <- fm_mesh_2d_inla(lattice = lattice, extend = FALSE, refine = FALSE)

# extract a part of the mesh inside a circle
    xy.in <- rowSums((mesh$loc[, 1:2] - 2)^2) < 1
    submesh <- excursions:::submesh.mesh(matrix(xy.in, nxy, nxy), mesh)
    plot(mesh$loc[, 1:2])
    lines(2 + cos(seq(0, 2 * pi, length.out = 100)), 2 + sin(seq(0, 2 * pi, length.out = 100)))
    plot(submesh, add = TRUE)
    points(mesh$loc[xy.in, 1:2], col = "2")
}

## End(Not run)</pre>
```

summary.excurobj 39

summary.excurobj

Summarise excurobj objects

### **Description**

Summary method for class "excurobj"

## Usage

```
## S3 method for class 'excurobj'
summary(object, ...)
## S3 method for class 'summary.excurobj'
print(x, ...)
## S3 method for class 'excurobj'
print(x, ...)
```

#### **Arguments**

object an object of class "excurobj", usually, a result of a call to excursions.

... further arguments passed to or from other methods.

x an object of class "summary.excurobj", usually, a result of a call to summary.excurobj.

tricontour

Calculate contour curves on a triangulation

## **Description**

Calculates contour curves and/or regions between them, for functions defined on a triangulation

# Usage

```
tricontour(
    x,
    z,
    nlevels = 10,
    levels = pretty(range(z, na.rm = TRUE), nlevels),
    ...
)

## S3 method for class 'inla.mesh'
tricontour(
    x,
    z,
```

```
nlevels = 10,
 levels = pretty(range(z, na.rm = TRUE), nlevels),
)
## S3 method for class 'matrix'
tricontour(
 х,
 Ζ,
 nlevels = 10,
 levels = pretty(range(z, na.rm = TRUE), nlevels),
 loc,
)
## S3 method for class 'list'
tricontour(
 Х,
 Ζ,
 nlevels = 10,
 levels = pretty(range(z, na.rm = TRUE), nlevels),
  type = c("+", "-"),
  tol = 1e-07,
)
tricontourmap(
 Х,
 Ζ,
 nlevels = 10,
 levels = pretty(range(z, na.rm = TRUE), nlevels),
)
## S3 method for class 'inla.mesh'
tricontourmap(
 х,
 z,
 nlevels = 10,
 levels = pretty(range(z, na.rm = TRUE), nlevels),
## S3 method for class 'matrix'
tricontourmap(
 Х,
  z,
```

# Arguments

| X       | An object generated by a call to inla.mesh.2d or inla.mesh.create, a triangle-vertex index matrix, or a list of triangulation information, list(loc, graph=list(tv)).           |
|---------|---|
| Z       | A vector containing the values to be contoured (NAs are allowed).   |
| nlevels | Number of contour levels desired, if and only if levels is not supplied.  |
| levels  | Numeric vector of levels at which to calculate contour lines.   |
|         | Additional arguments passed to the other methods.   |
| loc     | coordinate matrix, to be supplied when x is given as a triangle-vertex index matrix only.   |
| type    | "+" or "-", indicating positive or negative association. For +, the generated contours enclose regions where $u_1 \leq z < u_2$ , for - the regions fulfil $u_1 < z \leq u_2$ . |
| tol     | tolerance for determining if the value at a vertex lies on a level.   |
| output  | The format of the generated output. Implemented options are "sp" (default) and "fm" (and deprecated "inla.mesh.segment").   |

#### Value

For tricontour, a list with some of the fields that inla.mesh.segment objects have:

| loc | A coordinate matrix  |
|-----|--|
| idx | Contour segment indices, as a 2-column matrix, each row indexing a single segment  |
| grp | A vector of group labels. Each segment has a label, in 1,, nlevels*2+1, where even labels indicate interior on-level contour segments, and odd labels indicate boundary segments between levels. |

For tricontourmap, a list:

contour A list of sp or inla.mesh.segment objects defining countour curves (level sets)

A list of sp or inla.mesh.segment objects enclosing regions between level sets

## Author(s)

Finn Lindgren <finn.lindgren@gmail.com>

```
## Not run:
if (require("fmesher")) {
 ## Generate mesh and SPDE model
 n.lattice <- 20 # increase for more interesting, but slower, examples
 x \leftarrow seq(from = 0, to = 10, length.out = n.lattice)
 lattice <- fm_lattice_2d(x = x, y = x)
 mesh <- fm_rcdt_2d_inla(lattice = lattice, extend = FALSE, refine = FALSE)</pre>
 ## Generate an artificial sample
 sigma2.e <- 0.1
 n.obs <- 1000
 obs.loc <- cbind(</pre>
    runif(n.obs) * diff(range(x)) + min(x),
    runif(n.obs) * diff(range(x)) + min(x)
 Q <- fm_matern_precision(mesh, alpha = 2, rho = 3, sigma = 1)
 x <- fm_sample(n = 1, 0 = 0)
 A <- fm_basis(mesh, loc = obs.loc)
 Y <- as.vector(A %*% x + rnorm(n.obs) * sqrt(sigma2.e))
 ## Calculate posterior
 Q.post <- (Q + (t(A) %*% A) / sigma2.e)
 mu.post <- as.vector(solve(Q.post, (t(A) %*% Y) / sigma2.e))</pre>
 ## Calculate continuous contours
 tric <- tricontour(mesh,</pre>
   z = mu.post,
   levels = as.vector(quantile(x, c(0.25, 0.75)))
 ## Discrete domain contours
 map <- contourmap(</pre>
   n.levels = 2, mu = mu.post, Q = Q.post,
   alpha = 0.1, compute = list(F = FALSE), max.threads = 1
 ## Calculate continuous contour map
 setsc <- tricontourmap(mesh,</pre>
   z = mu.post,
   levels = as.vector(quantile(x, c(0.25, 0.75)))
 )
```

```
## Plot the results
reo <- mesh$idx$lattice
idx.setsc <- setdiff(names(setsc$map), "-1")
cols2 <- contourmap.colors(map,
    col = heat.colors(100, 0.5, rev = TRUE),
    credible.col = grey(0.5, 0)
)
names(cols2) <- as.character(-1:2)

par(mfrow = c(1, 2))
image(matrix(mu.post[reo], n.lattice, n.lattice),
    main = "mean", axes = FALSE, asp = 1
)
plot(setsc$map[idx.setsc], col = cols2[idx.setsc])
par(mfrow = c(1, 1))
}

## End(Not run)</pre>
```

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